

## Remarks on Hawking radiation as tunneling from the BTZ black holes

---

**Shuang-Qing Wu and Qing-Quan Jiang**

*College of Physical Science and Technology, Central China Normal University, Wuhan,  
Hubei 430079, People's Republic of China*

*E-mail: sqwu@phy.ccnu.edu.cn, jiangqingqua@126.com*

**ABSTRACT:** Hawking radiation viewed as a semiclassical tunneling process from the event horizon of the  $(2 + 1)$ -dimensional rotating BTZ black hole is carefully reexamined by taking into account not only the energy conservation but also the conservation of angular momentum when the effect of the emitted particle's self-gravitation is incorporated. In contrast to previous analysis of this issue in the literature, our result obtained here fits well to the Parikh-Wilczek's universal conclusion without any modification to the Bekenstein-Hawking area-entropy formulae of the BTZ black hole.

**KEYWORDS:** Classical Theories of Gravity, Black Holes, Black Holes in String Theory.

JHEP03(2006)079

Ever since Parikh and Wilczek [1] put forward a semiclassical framework to implement Hawking radiation as a tunneling process across the horizon of the static spherically symmetric black holes and demonstrated that the emission spectrum of black hole radiance is not strictly pure thermal, there have been considerable efforts to generalize this work to those of various spherically symmetric black holes [1–3], and all the obtained results are very successful to support the Parikh-Wilczek’s prescription. There also have some recent attempts [4] to extend this approach to the case of some stationary axisymmetric geometries, however, to the best of our knowledge, the treatments made in these researches [4] are not completely satisfactory until very recently we [5] present a plausible solution to this problem in the Kerr and Kerr-Newman black hole case.

As far as the case of a  $(2 + 1)$ -dimensional Bañados-Teitelboim-Zanelli (BTZ) [6] black hole is concerned, several investigations along this line have also been done [7–11]. For instance, Hawking radiation as a tunneling process from the charged and uncharged nonrotating BTZ black holes is discussed in ref. [7], while the spinning BTZ black hole case is investigated in [8, 9]. Vagenas [10] has also dealt with the case of the Achúcarro-Ortiz black hole [11] which is the Kaluza-Klein reduction of the  $(2 + 1)$ -dimensional BTZ model. We mention that a different methodology to apply the tunneling picture in the rotating BTZ black hole case has also been advised [12], which may be attributed to the complex path method [13].

In this note, we shall reexamine Hawking radiation as tunneling from the rotating BTZ black hole by taking into account not only the energy conservation but also the conservation of angular momentum and show that the Parikh-Wilczek’s universal relation can be preserved without modifying the Bekenstein-Hawking entropy expression of the BTZ black hole. This result is in contrast with previous analysis on the same subject [8], where the main result obtained there does not fit well to the universal one given by Parikh-Wilczek [1], but acquires a ‘self-gravitation’ correction to the Bekenstein-Hawking entropy formulae. We think the reason for this is that the rotation degree of freedom of the BTZ black hole had not been considered there. In addition, we shall revisit the same subject in a different choice of coordinate gauge also.

The BTZ black hole solution is an exact solution to Einstein field equation in a  $(2 + 1)$ -dimensional theory of gravity with a negative cosmological constant ( $\Lambda = -1/l^2$ ). The corresponding line element is

$$ds^2 = -\Delta dt^2 + \frac{dr^2}{\Delta} + r^2 \left( d\phi - \frac{J}{2r^2} dt \right)^2, \quad (1)$$

where the lapse function

$$\Delta = -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}, \quad (2)$$

in which two integration constants  $M$  and  $J$  denote the ADM mass and angular momentum of the BTZ black hole, respectively. (The BTZ unit  $G_3 = 1/8$  is adopted through out this paper.)

The outer (event) and inner horizons are given by the condition  $\Delta = 0$ , and read

$$r_{\pm}^2 = l^2 \frac{M \pm \sqrt{M^2 - J^2/l^2}}{2}. \quad (3)$$

The Bekenstein-Hawking entropy of the spinning BTZ black holes is twice the perimeter  $L$  of the event horizon

$$S(M, J, l) = 2L = 4\pi r_+, \quad (4)$$

the surface gravity and angular velocity at the event (outer) horizon are easily evaluated

$$\kappa = \frac{1}{2} \frac{d\Delta}{dr} \Big|_{r=r_+} = \frac{r_+^2 - r_-^2}{r_+ l^2}, \quad \Omega_+ = - \frac{g_{t\phi}}{g_{\phi\phi}} \Big|_{r=r_+} = \frac{J}{2r_+^2} = \frac{r_-}{r_+ l}. \quad (5)$$

The idea of Parikh-Wilczek's method is to describe the BTZ black hole background as dynamical by treating the Hawking radiation as a tunneling process. In order to apply this methodology to the case of a rotating BTZ black hole and to do an explicit computation of the tunneling rate, a key trick is to introduce a Painlevé-type coordinate system that is well-behaved at the event (outer) horizon. There are essentially two slightly different approaches to arrive at this aim. The first is that one must firstly find a Painlevé-BTZ coordinate system and then transform it into the dragging coordinate system. This method is well addressed by us in ref. [5], where the reason why one must adopt a dragging coordinate system is also explained in details there. Vagenas [8] only considered the energy conservation but no angular momentum conservation, i.e., he viewed the ADM mass of the BTZ black hole can fluctuate but the angular momentum  $J$  is kept fixed. [Note that the total ADM mass is kept fixed while the mass of the BTZ black hole decreases due to the emitted radiation.] He only performed the first step without further transforming it into the dragging coordinate system in his analysis; at a cost, he had to modify the well-known Bekenstein-Hawking area-entropy formulae of the BTZ black hole. The second one is to reverse the order of the above two steps. That is, one firstly transforms the metric (1) into the dragging coordinate system and then seeks a Painlevé-type coordinate transformation. In the former step of this method, the resultant metric [9, 10] for the spinning BTZ black hole is (after performing a dragging coordinate transformation  $d\phi = J/(2r^2)dt$ )

$$ds^2 = -\Delta dt^2 + \frac{dr^2}{\Delta}, \quad (6)$$

which represents a (1 + 1)-dimensional hypersurface in the (2 + 1)-dimensional BTZ spacetime. This line element (6) is, in fact, identified with that of the Achúcarro-Ortiz black hole [11] which is the Kaluza-Klein reduction of the (2 + 1)-dimensional BTZ black hole. The subsequent tunneling analysis [9] essentially reproduces those of a two-dimensional dilatonic black hole [10].

It should be pointed out that the second method is only effective in the 3-dimensional case unlike the first one which is universal in any dimensions. In what follows, we shall adopt the first method. For this purpose, we introduce the time coordinate  $\tau$  and the angular coordinate  $\varphi$  by imposing the ansatz:

$$dt = d\tau - \frac{\sqrt{1-F}}{\Delta} dr, \quad (7)$$

$$d\phi = d\varphi - \left(\frac{J}{2r^2}\right) \frac{\sqrt{1-F}}{\Delta} dr, \quad (8)$$

where we choose the function  $F = \Delta$  as Vagenas did in ref. [8].

Obviously the above transformations (7), (8) eliminate the coordinate singularities at the horizons  $r_{\pm}$ . The line element (1) is now written as a Painlevé-type form

$$\begin{aligned} ds^2 &= -\Delta d\tau^2 + 2\sqrt{1-\Delta}d\tau dr + dr^2 + r^2\left(d\varphi - \frac{J}{2r^2}d\tau\right)^2 \\ &= -d\tau^2 + (\sqrt{1-\Delta}d\tau + dr)^2 + r^2\left(d\varphi - \frac{J}{2r^2}d\tau\right)^2. \end{aligned} \quad (9)$$

As explained in ref. [5], there are two essential reasons that one must further transform the above Painlevé-BTZ metric (9) into a dragging coordinate system; The first reason is that just as the original BTZ line element (1) the infinite red-shift surface  $r_{TLS} = l\sqrt{M}$  does not coincide with the outer horizon  $r_+$  so that the geometrical optical limit cannot be used since the tunneling computation is essentially a kind of WKB (s-wave) approximation. The other is that one must consider the dragging effect in a rotating black hole spacetime since the physical field must be dragged also in a rotating background spacetime. So a physically reasonable description must be within a dragging coordinate system, which can be equivalently to state that an observer is rest at such a reference system. Moreover, by performing a dragging coordinate transformation  $d\varphi = J/(2r^2)d\tau$ , the infinite red-shift surface coincides with the event horizon in the new line element which shall be referred to as a dragged Painlevé-BTZ metric

$$d\hat{s}^2 = -\Delta d\tau^2 + 2\sqrt{1-\Delta}d\tau dr + dr^2 = -d\tau^2 + (\sqrt{1-\Delta}d\tau + dr)^2, \quad (10)$$

so that the WKB approximation can be applied now.

It is easily to observe that the above metric (10) can also be deduced from the line element of a Achúcarro-Oritz black hole [11] by carrying out only the temporal coordinate transformation (7). This implies that the dragged Painlevé-BTZ (10) can also be obtained by means of the second method mentioned above. The reason why two different procedures lead to the same result is that the transformations (7), (8) keep the dragging angular velocity  $\Omega$  unchanged

$$\Omega = \frac{d\phi}{dt} = \frac{d\varphi}{d\tau} = \frac{J}{2r^2}. \quad (11)$$

However it should be stressed that this coincidence only occurs in the three-dimensional case, in higher dimensions two different methods in general derive two distinct line elements.

To apply Parikh-Wilczek's semiclassical tunneling analysis, the radial null ( $d\hat{s}^2 = 0$ ) geodesics followed by a massless particle need to be determined as follows

$$\dot{r} \equiv \frac{dr}{d\tau} = \pm 1 - \sqrt{1-\Delta}, \quad (12)$$

where the signs  $+$  and  $-$  correspond to the outgoing and ingoing geodesics, respectively, under the assumption that  $\tau$  increases towards future.

Let us now focus on a semiclassical treatment of the associated radiation (outgoing massless particle). We adopt the picture of a pair of virtual particles spontaneously created just inside the event horizon. The positive energy particle tunnels out from the outer horizon while the negative partner is absorbed by the black hole resulting in a decrease in the mass. In the case of the rotating BTZ black hole with fixed angular momentum  $J$ , the

emitted particle is simply visualized as a shell of energy (mass)  $\omega$ . Taking into account the energy conservation, we must fix the total ADM mass and let the ADM mass  $M$  of the BTZ black hole vary. If a shell of energy  $\omega$  is radiated outwards the outer horizon, then the BTZ black hole mass will be reduced to  $M - \omega$ , so the line element will be modified to

$$d\hat{s}^2 = -\tilde{\Delta}d\tau^2 + 2\sqrt{1 - \tilde{\Delta}}d\tau dr + dr^2, \tag{13}$$

and the shell will accordingly travel on the modified geodesics

$$\dot{r} = 1 - \sqrt{1 - \tilde{\Delta}} = \frac{\tilde{\Delta}}{1 + \sqrt{1 - \tilde{\Delta}}}, \tag{14}$$

where the lapse function  $\Delta$  is modified to  $\tilde{\Delta}$  with the replacement of mass  $M$  by  $M - \omega$ ,

$$\tilde{\Delta} = -(M - \omega) + \frac{r^2}{l^2} + \frac{J^2}{4r^2}. \tag{15}$$

Since the emission rate from a spinning BTZ black hole can be expressed in terms of the imaginary part of the action for an outgoing positive-energy particle as

$$\Gamma = e^{-2\text{Im} S}, \tag{16}$$

we therefore need to evaluate the imaginary part of the action

$$\text{Im} S = \text{Im} \int_{r_{\text{in}}}^{r_{\text{out}}} p_r dr = \text{Im} \int_{r_{\text{in}}}^{r_{\text{out}}} \int_0^{p_r} dp'_r dr, \tag{17}$$

for such a particle which crosses the event horizon outwards from  $r_{\text{in}}$  to  $r_{\text{out}}$  (Note that:  $r_{\text{in}} = r_+ > r_{\text{out}} = \tilde{r}_+$ ), where

$$r_{\text{out}}^2 = l^2 \frac{(M - \omega) + \sqrt{(M - \omega)^2 - J^2/l^2}}{2}. \tag{18}$$

The transition from the momentum variable to the energy variable can be made by using Hamilton's equation of motion

$$\dot{r} = \frac{dH}{dp_r} = \frac{d(M - \omega)}{dp_r}, \tag{19}$$

and eq. (14). After some calculations we get the explicit result

$$\text{Im} S = \text{Im} \int_{r_{\text{in}}}^{r_{\text{out}}} \int_M^{M-\omega} \frac{d(M - \omega') dr}{1 - \sqrt{1 + (M - \omega') - \frac{r^2}{l^2} - \frac{J^2}{4r^2}}} = 2\pi(r_{\text{in}} - r_{\text{out}}). \tag{20}$$

Apparently the emission probability depends not only on the mass  $M$  and angular momentum  $J$  of the BTZ black hole but also on the energy  $\omega$  of the emitted massless particle

$$\Gamma = e^{-2\text{Im} S} = e^{4\pi(r_{\text{out}} - r_{\text{in}})} = e^{\Delta S}, \tag{21}$$

where  $\Delta S = S(M - \omega, J, l) - S(M, J, l)$  is the change in the entropy of the BTZ black hole before and after the emission of the shell of energy  $\omega$ .

We see that the tunneling rate fits well to the universal result obtained by Parikh and Wilczek [1] in the Schwarzschild and Reissner-Nordström black hole case, without modifying the standard area-entropy expression of a rotating BTZ black hole. This is in contrast with the previous result obtained for a spinning BTZ black hole with a fixed  $J$  [10]. We deduce that the entropy of the  $(2 + 1)$ -dimensional BTZ black hole is still proportional to its horizon area, therefore, it is clear that previous modification to the Bekenstein-Hawking area-entropy formulae for the BTZ black hole is not necessary [9] although the emitted particle's self-gravitation effect is incorporated.

However, it should be noted that the preceding discussion is limited to the case of energy conservation only. Since we are considering a spinning BTZ black hole, so the rotation degree of freedom should be well addressed also. This can be compensated by treating the emitted massless particle as a shell of energy  $\omega$  and angular momentum  $j$ . Now taking into account not only the energy conservation but also the conservation of angular momentum, we must fix the total ADM mass and total angular momentum, but let the ADM mass  $M$  and angular momentum  $J$  of the BTZ black hole vary. If a shell of energy  $\omega$  and angular momentum  $j$  tunnels out from the outer horizon, then the BTZ black hole mass and angular momentum will be reduced to  $M - \omega$  and  $J - j$ , respectively. Therefore the modified line element and modified radial null geodesics are still respectively represented by eqs. (13) and (14) with the lapse function  $\Delta$  replaced by

$$\tilde{\Delta} = -(M - \omega) + \frac{r^2}{l^2} + \frac{(J - j)^2}{4r^2}. \tag{22}$$

It follows that the evaluation of the imaginary part of the action is of no difficulty to complete by eliminating a cyclic coordinate  $\varphi$  in the Lagrangian function and goes as follows

$$\begin{aligned} \text{Im } S &= \text{Im} \left[ \int_{r_{\text{in}}}^{r_{\text{out}}} p_r dr - \int_{\varphi_{\text{in}}}^{\varphi_{\text{out}}} p_\varphi d\varphi \right] = \text{Im} \int_{r_{\text{in}}}^{r_{\text{out}}} \left[ \int_0^{p_r} dp'_r dr - \int_0^{p_\varphi} dp'_\varphi d\varphi \right] \\ &= \text{Im} \int_{r_{\text{in}}}^{r_{\text{out}}} \left[ \int_{(0,0)}^{(p_r, p_\varphi)} \left( \dot{r} dp'_r - \dot{\varphi} dp'_\varphi \right) \right] \frac{dr}{\dot{r}}, \end{aligned} \tag{23}$$

where  $p_r$  and  $p_\varphi$  are two canonical momenta conjugate to the coordinates  $r$  and  $\varphi$ , respectively. As before,  $r_{\text{in}} = r_+$  and  $r_{\text{out}} = \tilde{r}_+$ , which is given by

$$r_{\text{out}}^2 = l^2 \frac{(M - \omega) + \sqrt{(M - \omega)^2 - (J - j)^2/l^2}}{2} = \tilde{r}_+^2, \tag{24}$$

are the locations of the event horizon before and after a particle tunnels out.

To remove the momentum in favor of energy, we can make use of the Hamilton's equations

$$\begin{aligned} \dot{r} &= \left. \frac{dH}{dp_r} \right|_{(r; \varphi, p_\varphi)} = \frac{d(M - \omega)}{dp_r}, \\ \dot{\varphi} &= \left. \frac{dH}{dp_\varphi} \right|_{(\varphi; r, p_r)} = \tilde{\Omega} \frac{d(J - j)}{dp_\varphi}, \end{aligned} \tag{25}$$

where  $dH_{(\varphi;r,p_r)} = \tilde{\Omega}d(J-j)$  represents the energy change of the black hole because of the loss of angular momentum when a particle tunnels out, and the dragging angular velocity is given by

$$\tilde{\Omega} = \frac{J-j}{2r^2}. \tag{26}$$

Now substituting eqs. (14) and (25) into (23) and interchanging the order of integration yields

$$\begin{aligned} \text{Im } S &= \text{Im} \int_{r_{\text{in}}}^{r_{\text{out}}} \int_{(M,J)}^{(M-\omega,J-j)} \left[ d(M-\omega') - \tilde{\Omega}d(J-j') \right] \frac{dr}{\dot{r}} \\ &= \text{Im} \int_{(M,J)}^{(M-\omega,J-j)} \int_{r_{\text{in}}}^{r_{\text{out}}} \frac{1 + \sqrt{1 - \tilde{\Delta}}}{\tilde{\Delta}} \left[ d(M-\omega') - \frac{(J-j')}{2r^2}d(J-j') \right] dr \\ &= 2\pi(r_{\text{in}} - r_{\text{out}}), \end{aligned} \tag{27}$$

thus the tunneling probability

$$\Gamma = e^{-2\text{Im } S} = e^{4\pi(r_{\text{out}}-r_{\text{in}})} = e^{\Delta S}, \tag{28}$$

reproduces the Parikh-Wilczek's standard result for the tunneling picture, where  $\Delta S = S(M-\omega, J-j, l) - S(M, J, l)$  is the change in the entropy of the BTZ black hole before and after the particle emission. Again we see that there is no need to modify the well-known Bekenstein-Hawking area-entropy relation of a spinning BTZ black hole even when we incorporate the particle's self-gravitation effect.

In what follows, we would like to point out that the ansatz for the coordinate transformations (7), (8) with the choice of the function  $F = \Delta$  is not unique. Alternatively we can make a different choice of the function

$$F = \frac{\Delta}{\Delta_0}, \quad \Delta_0 = \Delta|_{M=0} = \frac{r^2}{l^2} + \frac{J^2}{4r^2}. \tag{29}$$

With this choice for the coordinate transformations (7), (8), the line element (1) is now transformed into the form

$$\begin{aligned} ds^2 &= -\Delta d\tau^2 + 2\sqrt{1 - \frac{\Delta}{\Delta_0}}d\tau dr + \frac{dr^2}{\Delta_0} + r^2 \left( d\varphi - \frac{J}{2r^2}d\tau \right)^2 \\ &= -\Delta_0 d\tau^2 + \left( \sqrt{\Delta_0 - \Delta}d\tau + \frac{dr}{\sqrt{\Delta_0}} \right)^2 + r^2 \left( d\varphi - \frac{J}{2r^2}d\tau \right)^2. \end{aligned} \tag{30}$$

Similarly the dragging coordinate transformation  $d\varphi = J/(2r^2)d\tau$  further reduces the above line element to

$$d\hat{s}^2 = -\Delta d\tau^2 + 2\sqrt{1 - \frac{\Delta}{\Delta_0}}d\tau dr + \frac{dr^2}{\Delta_0} = -\Delta_0 d\tau^2 + \frac{1}{\Delta_0} \left( \sqrt{1 - \frac{\Delta}{\Delta_0}}d\tau + dr \right)^2, \tag{31}$$

from which the radial null ( $d\hat{s}^2 = 0$ ) geodesics followed by a massless particle are determined

$$\dot{r} = \Delta_0 \left( \pm 1 - \sqrt{1 - \Delta/\Delta_0} \right). \tag{32}$$

Incidentally we point out that the line element (31) can also be derived by using the second approach mentioned above.

Now the explicit tunneling computation completely repeats the preceding procedure with the emitted particle being viewed as a shell of energy  $\omega$  and angular momentum  $j$ . By considering the particle's self-gravitation effect and the conservation of energy as well as that of angular momentum, when a shell of energy  $\omega$  and angular momentum  $j$  is radiated outwards the outer horizon, then the ADM mass  $M$  and angular momentum  $J$  of the BTZ black hole will be reduced to  $M - \omega$  and  $J - j$ , respectively. Accordingly the line element and radial null geodesics will be modified as

$$d\hat{s}^2 = -\tilde{\Delta}d\tau^2 + 2\sqrt{1 - \tilde{\Delta}/\tilde{\Delta}_0}d\tau dr + \frac{dr^2}{\tilde{\Delta}_0}, \quad (33)$$

$$\dot{r} = \tilde{\Delta}_0 \left(1 - \sqrt{1 - \tilde{\Delta}/\tilde{\Delta}_0}\right) = \frac{\tilde{\Delta}}{1 + \sqrt{1 - \tilde{\Delta}/\tilde{\Delta}_0}}, \quad (34)$$

where

$$\tilde{\Delta} = -(M - \omega) + \frac{r^2}{l^2} + \frac{(J - j)^2}{4r^2}, \quad \tilde{\Delta}_0 = \frac{r^2}{l^2} + \frac{(J - j)^2}{4r^2}. \quad (35)$$

Note that near the outer horizon  $r \approx \tilde{r}_+$ , the modified radial null geodesic has the asymptotic behavior

$$\dot{r} \approx \tilde{\kappa}(r - \tilde{r}_+), \quad \tilde{\kappa} = \frac{1}{2} \frac{d\tilde{\Delta}}{dr} \Big|_{r=\tilde{r}_+}, \quad (36)$$

and the dragging angular velocity takes its value at the event horizon

$$\tilde{\Omega}_+ = \frac{J - j}{2\tilde{r}_+^2}, \quad (37)$$

so it is easily to see that the imaginary part of the action is dominated by its value at the outer horizon

$$\begin{aligned} \text{Im } S &= \text{Im} \int_{r_{\text{in}}}^{r_{\text{out}}} \int_{(M,J)}^{(M-\omega, J-j)} \left[ d(M - \omega') - \tilde{\Omega} d(J - j') \right] \frac{dr}{\dot{r}} \\ &\approx \text{Im} \int_{(M,J)}^{(M-\omega, J-j)} \int_{r_{\text{in}}}^{r_{\text{out}}} \left[ d(M - \omega') - \tilde{\Omega}_+ d(J - j') \right] \frac{dr}{\tilde{\kappa}(r - \tilde{r}_+)} \\ &= - \int_{(M,J)}^{(M-\omega, J-j)} \frac{\pi}{\tilde{\kappa}} \left[ d(M - \omega') - \frac{(J - j')}{2\tilde{r}_+^2} d(J - j') \right]. \end{aligned} \quad (38)$$

On the other hand, from the event horizon equation

$$\tilde{\Delta}|_{r=\tilde{r}_+} = -(M - \omega) + \frac{\tilde{r}_+^2}{l^2} + \frac{(J - j)^2}{4\tilde{r}_+^2} = 0, \quad (39)$$

one can use a simple algebra approach to prove the differential mass formulae [14] of the first law of a spinning BTZ black hole

$$d(M - \omega) - \frac{(J - j)}{2\tilde{r}_+^2} d(J - j) = \frac{d\tilde{\Delta}}{dr} \Big|_{r=\tilde{r}_+} d\tilde{r}_+ = 2\tilde{\kappa}d\tilde{r}_+. \quad (40)$$



Inserting the identity (40) into the integral (38), the final result is very concise

$$\text{Im } S = -2\pi \int_{r_{\text{in}}}^{r_{\text{out}}} d\tilde{r}'_+ = 2\pi(r_{\text{in}} - r_{\text{out}}) = -\frac{1}{2}\Delta S, \quad (41)$$

and restores the standard expression for the tunneling rate.

To summarize, we have revisited Hawking radiation as a semiclassical tunneling process from the outer horizon of a rotating BTZ black hole by taking into account the energy conservation and the conservation of angular momentum. The Parikh-Wilczek's universal conclusion can be retained without modifying the well-known Bekenstein-Hawking area-entropy formulae of the BTZ black hole. This result is in contrast with previous analysis of the same subject in the literature. Besides, we have also shown that the so-called Painlevé-type coordinate transformation is not unique, two different choices of the coordinate transformation lead to the same result. This demonstrates that our discussion present here is consistent on its behalf.

## Acknowledgments

S.-Q. Wu was supported by a starting fund from Central China Normal University and by the Natural Science Foundation of China.

## References

- [1] M.K. Parikh and F. Wilczek, *Hawking radiation as tunneling*, *Phys. Rev. Lett.* **85** (2000) 5042 [[hep-th/9907001](#)];  
M.K. Parikh, *New coordinates for de Sitter space and de Sitter radiation*, *Phys. Lett.* **B 546** (2002) 189 [[hep-th/0204107](#)]; *A secret tunnel through the horizon*, *Int. J. Mod. Phys.* **D 13** (2004) 2351 [*Gen. Rel. Grav.* **36** (2004) 2419] [[hep-th/0405160](#)]; *Energy conservation and Hawking radiation*, [hep-th/0402166](#).
- [2] S. Hemming and E. Keski-Vakkuri, *Hawking radiation from AdS black holes*, *Phys. Rev.* **D 64** (2001) 044006 [[gr-qc/0005115](#)];  
A.J.M. Medved, *Radiation via tunneling from a de Sitter cosmological horizon*, *Phys. Rev.* **D 66** (2002) 124009 [[hep-th/0207247](#)];  
E.C. Vagenas, *Generalization of the KKW analysis for black hole radiation*, *Phys. Lett.* **B 559** (2003) 65 [[hep-th/0209185](#)];  
M. Arzano, A.J.M. Medved and E.C. Vagenas, *Hawking radiation as tunneling through the quantum horizon*, *JHEP* **09** (2005) 037 [[hep-th/0505266](#)];  
M. Arzano, *Tunneling through the quantum horizon*, *Mod. Phys. Lett.* **A 21** (2006) 41 [[hep-th/0504188](#)];  
A.J.M. Medved and E.C. Vagenas, *On Hawking radiation as tunneling with back-reaction*, *Mod. Phys. Lett.* **A 20** (2005) 2449 [[gr-qc/0504113](#)];  
I. Radinschi, *The KKW generalized analysis for a magnetic stringy black hole*, [gr-qc/0412111](#).
- [3] Q.-Q. Jiang and S.-Q. Wu, *Hawking radiation of charged particles as tunneling from Reissner-Nordström-de Sitter black holes with a global monopole*, *Phys. Lett.* **B 635** (2006) 151 [[hep-th/0511123](#)];

- J.-Y. Zhang and Z. Zhao, *Hawking radiation of charged particles via tunneling from the Reissner-Nordström black hole*, *JHEP* **10** (2005) 055;
- J.-Y. Zhang and Z. Zhao, *Massive particles' black hole tunneling and de Sitter tunneling*, *Nucl. Phys.* **B 725** (2005) 173;
- Y.W. Han and S.Z. Yang, *Quantum tunnelling and Hawking radiation of Schwarzschild-anti-de Sitter black hole with topological defect*, *Chin. Phys. Lett.* **22** (2005) 2769;
- J. Ren, Z. Zhao and C.J. Gao, *Hawking radiation via tunnelling from arbitrarily dimensional Schwarzschild black holes*, *Chin. Phys. Lett.* **22** (2005) 2489;
- J. Ren and Z. Zhao, *Tunnelling effect from a Vaidya-de Sitter black hole*, *Chin. Phys.* **15** (2006) 292;
- H.Z. Fang, Y.P. Hu and Z. Zhao, *Hawking radiation from the Horowitz-Strominger black hole*, *Chin. Phys. Lett.* **22** (2005) 1611;
- H.Z. Fang, J. Ren and Z. Zhao, *Particle tunnels from Garfinkle-Horne black hole and Horowitz-Strominger black hole*, *Int. J. Mod. Phys.* **D 14** (2005) 1699;
- W. Liu, *Hawking radiation as tunnelling in static black holes*, [gr-qc/0512098](#).
- [4] J.-Y. Zhang and Z. Zhao, *Hawking radiation via tunneling from Kerr black holes*, *Mod. Phys. Lett.* **A 20** (2005) 1673; *New coordinates for Kerr-Newman black hole radiation*, *Phys. Lett.* **B 618** (2005) 14;
- S.Z. Yang, *Kerr-Newman-Kasuya black hole tunnelling radiation*, *Chin. Phys. Lett.* **22** (2005) 2492;
- S.Z. Yang, Q.Q. Jiang and H.L. Li, *Quantum tunnelling radiation of Einstein-Maxwell-Dilaton-Axion black hole*, *Chin. Phys.* **14** (2005) 2411;
- J. Zhang and Z. Zhao, *Charged particles' tunnelling from the Kerr-Newman black hole*, [gr-qc/0512153](#).
- [5] Q.Q. Jiang and S.Q. Wu, *New form of the Kerr solution and its tunneling radiation*, *Acta Physica Sinica* **55** (2006) to appear (in Chinese);
- Q.-Q. Jiang, S.-Q. Wu and X. Cai, *Hawking radiation as tunneling from the Kerr and Kerr-Newman black holes*, *Phys. Rev.* **D 73** (2006) 064003 [[hep-th/0512351](#)], erratum *ibid.* **D73** (2006) 069902.
- [6] M. Bañados, C. Teitelboim and J. Zanelli, *The black hole in three-dimensional space-time*, *Phys. Rev. Lett.* **69** (1992) 1849 [[hep-th/9204099](#)];
- M. Bañados, M. Henneaux, C. Teitelboim and J. Zanelli, *Geometry of the (2+1) black hole*, *Phys. Rev.* **D 48** (1993) 1506 [[gr-qc/9302012](#)];
- S. Carlip, *The (2+1)-dimensional black hole*, *Class. and Quant. Grav.* **12** (1995) 2853 [[gr-qc/9506079](#)].
- [7] A.J.M. Medved, *Radiation via tunneling in the charged BTZ black hole*, *Class. and Quant. Grav.* **19** (2002) 589 [[hep-th/0110289](#)];
- M. Hortacsu, H.T. Ozcelik and B. Yapiskan, *Properties of solutions in 2+1 dimensions*, *Gen. Rel. Grav.* **35** (2003) 1209 [[gr-qc/0302005](#)].
- [8] E.C. Vagenas, *Quantum corrections to the Bekenstein-Hawking entropy of the BTZ black hole via self-gravitation*, *Phys. Lett.* **B 533** (2002) 302 [[hep-th/0109108](#)].
- [9] W. Liu, *New coordinates for BTZ black hole and Hawking radiation via tunnelling*, *Phys. Lett.* **B 634** (2006) 541 [[gr-qc/0512099](#)].
- [10] E.C. Vagenas, *Two-dimensional dilatonic black holes and Hawking radiation*, *Mod. Phys. Lett.* **A 17** (2002) 609 [[hep-th/0108147](#)].

- [11] A. Achúcarro and M.E. Ortiz, *Relating black holes in two and three dimensions*, *Phys. Rev. D* **48** (1993) 3600 [[hep-th/9304068](#)];  
D.A. Lowe and A. Strominger, *Exact four-dimensional dyonic black holes and Bertotti-Robinson space-times in string theory*, *Phys. Rev. Lett.* **73** (1994) 1468 [[hep-th/9403186](#)].
- [12] M. Angheben, M. Nadalini, L. Vanzo and S. Zerbini, *Hawking radiation as tunneling for extremal and rotating black holes*, *JHEP* **05** (2005) 014 [[hep-th/0503081](#)].
- [13] K. Srinivasan and T. Padmanabhan, *Particle production and complex path analysis*, *Phys. Rev. D* **60** (1999) 024007 [[gr-qc/9812028](#)];  
S. Shankaranarayanan, K. Srinivasan and T. Padmanabhan, *Method of complex paths and general covariance of Hawking radiation*, *Mod. Phys. Lett. A* **16** (2001) 571 [[gr-qc/0007022](#)];  
S. Shankaranarayanan, T. Padmanabhan and K. Srinivasan, *Hawking radiation in different coordinate settings: complex paths approach*, *Class. and Quant. Grav.* **19** (2002) 2671 [[gr-qc/0010042](#)];  
T. Padmanabhan, *Entropy of horizons, complex paths and quantum tunneling*, *Mod. Phys. Lett. A* **19** (2004) 2637 [[gr-qc/0405072](#)].
- [14] S. Wang, S.Q. Wu, F. Xie and L. Dan, *The first laws of thermodynamics of the (2+1)-dimensional BTZ black holes and Kerr-de Sitter spacetimes*, [hep-th/0601147](#).